1 Concepts of Motion

Motion takes many forms. The ski jumper seen here is an example of translational motion.

IN THIS CHAPTER, you will learn the fundamental concepts of motion.

What is a chapter preview?

Each chapter starts with an overview. Think of it as a roadmap to help you get oriented and make the most of your studying.

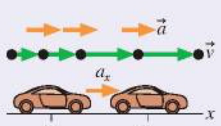
« LOOKING BACK A Looking Back reference tells you what material from previous chapters is especially important for understanding the new topics. A quick review will help your learning. You will find additional Looking Back references within the chapter, right at the point they're needed.

What is motion?

Before solving motion problems, we must learn to *describe* motion. We will use

- Motion diagrams
- Graphs
- Pictures

Motion concepts introduced in this chapter include position, velocity, and acceleration.



Known

$x_0 = v_{0x} = t_0 = 0$

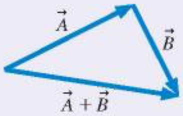
$a_x = 2.0\ \text{m/s}^2$

Find

x_1

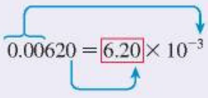
Why do we need vectors?

Many of the quantities used to describe motion, such as velocity, have both a size and a direction. We use vectors to represent these quantities. This chapter introduces graphical techniques to add and subtract vectors. Chapter 3 will explore vectors in more detail.



Why are units and significant figures important?

Scientists and engineers must communicate their ideas to others. To do so, we have to agree about the *units* in which quantities are measured. In physics we use metric units, called SI units. We also need rules for telling others how accurately a quantity is known. You will learn the rules for using significant figures correctly.



Why is motion important?

The universe is in motion, from the smallest scale of electrons and atoms to the largest scale of entire galaxies. We'll start with the motion of everyday objects, such as cars and balls and people. Later we'll study the motions of waves, of atoms in gases, and of electrons in circuits. Motion is the one theme that will be with us from the first chapter to the last.

1.1 Motion Diagrams

Motion is a theme that will appear in one form or another throughout this entire book. Although we all have intuition about motion, based on our experiences, some of the important aspects of motion turn out to be rather subtle. So rather than jumping immediately into a lot of mathematics and calculations, this first chapter focuses on *visualizing* motion and becoming familiar with the *concepts* needed to describe a moving object. Our goal is to lay the foundations for understanding motion.

FIGURE 1.1 Four basic types of motion.

Linear motion

Circular motion

Projectile motion

Rotational motion

To begin, let's define **motion** as the change of an object's position with time. **FIGURE 1.1** shows four basic types of motion that we will study in this book. The first three—linear, circular, and projectile motion—in which the object moves through space are called **translational motion.** The path along which the object moves, whether straight or curved, is called the object's **trajectory.** Rotational motion is somewhat different because there's movement but the object as a whole doesn't change position. We'll defer rotational motion until later and, for now, focus on translational motion.

FIGURE 1.2 Four frames from a video.

Making a Motion Diagram

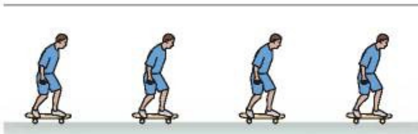
An easy way to study motion is to make a video of a moving object. A video camera, as you probably know, takes images at a fixed rate, typically 30 every second. Each separate image is called a *frame*. As an example, **FIGURE 1.2** shows four frames from a video of a car going past. Not surprisingly, the car is in a somewhat different position in each frame.

Suppose we edit the video by layering the frames on top of each other, creating the composite image shown in **FIGURE 1.3**. This edited image, showing an object's position at several *equally spaced instants of time,* is called a **motion diagram.** As the examples below show, we can define concepts such as constant speed, speeding up, and slowing down in terms of how an object appears in a motion diagram.

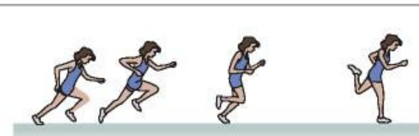
FIGURE 1.3 A motion diagram of the car shows all the frames simultaneously.

NOTE It's important to keep the camera in a *fixed position* as the object moves by. Don't "pan" it to track the moving object.

Examples of motion diagrams



Images that are *equally spaced* indicate an object moving with *constant speed.*



An *increasing distance* between the images shows that the object is *speeding up.*

A *decreasing distance* between the images shows that the object is *slowing down.*

STOP TO THINK 1.1 Which car is going faster, A or B? Assume there are equal intervals of time between the frames of both videos.

NOTE Each chapter will have several *Stop to Think* questions. These questions are designed to see if you've understood the basic ideas that have been presented. The answers are given at the end of the book, but you should make a serious effort to think about these questions before turning to the answers.

We can model an airplane's takeoff as a particle (a descriptive model) undergoing constant acceleration (a descriptive model) in response to constant forces (an explanatory model).

1.2 Models and Modeling

The real world is messy and complicated. Our goal in physics is to brush aside many of the real-world details in order to discern patterns that occur over and over. For example, a swinging pendulum, a vibrating guitar string, a sound wave, and jiggling atoms in a crystal are all very different—yet perhaps not so different. Each is an example of a system moving back and forth around an equilibrium position. If we focus on understanding a very simple oscillating system, such as a mass on a spring, we'll automatically understand quite a bit about the many real-world manifestations of oscillations.

Stripping away the details to focus on essential features is a process called *modeling.* A **model** is a highly simplified picture of reality, but one that still captures the essence of what we want to study. Thus "mass on a spring" is a simple but realistic model of almost all oscillating systems.

Models allow us to make sense of complex situations by providing a framework for thinking about them. One could go so far as to say that developing and testing models is at the heart of the scientific process. Albert Einstein once said, "Physics should be as simple as possible—but not simpler." We want to find the simplest model that allows us to understand the phenomenon we're studying, but we can't make the model so simple that key aspects of the phenomenon get lost.

We'll develop and use many models throughout this textbook; they'll be one of our most important thinking tools. These models will be of two types:

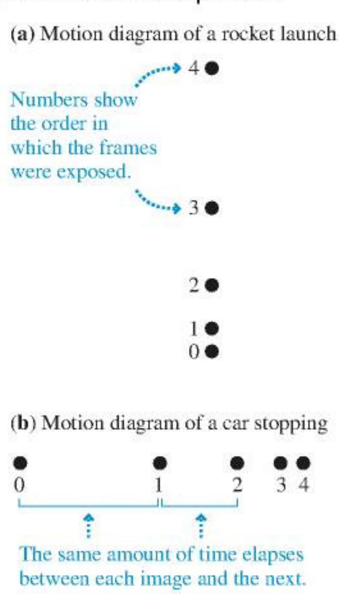
- *Descriptive models:* What are the essential characteristics and properties of a phenomenon? How do we describe it in the simplest possible terms? For example, the mass-on-a-spring model of an oscillating system is a descriptive model.
- *Explanatory models:* Why do things happen as they do? Explanatory models, based on the laws of physics, have predictive power, allowing us to test—against experimental data—whether a model provides an adequate explanation of our observations.

The Particle Model

For many types of motion, such as that of balls, cars, and rockets, the motion of the object *as a whole* is not influenced by the details of the object's size and shape. All we really need to keep track of is the motion of a single point on the object, so we can treat the object *as if* all its mass were concentrated into this single point. An object that can be represented as a mass at a single point in space is called a **particle.** A particle has no size, no shape, and no distinction between top and bottom or between front and back.

If we model an object as a particle, we can represent the object in each frame of a motion diagram as a simple dot rather than having to draw a full picture. **FIGURE 1.4** shows how much simpler motion diagrams appear when the object is represented as a particle. Note that the dots have been numbered 0, 1, 2, . . . to tell the sequence in which the frames were exposed.

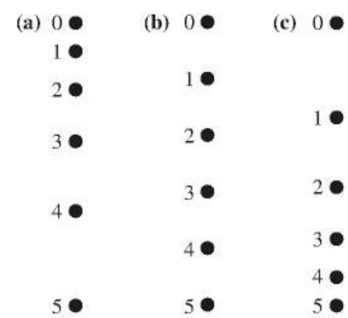
FIGURE 1.4 Motion diagrams in which the object is modeled as a particle.



Treating an object as a particle is, of course, a simplification of reality—but that's what modeling is all about. The **particle model** of motion is a simplification in which we treat a moving object as if all of its mass were concentrated at a single point. The particle model is an excellent approximation of reality for the translational motion of cars, planes, rockets, and similar objects.

Of course, not everything can be modeled as a particle; models have their limits. Consider, for example, a rotating gear. The center doesn't move at all while each tooth is moving in a different direction. We'll need to develop new models when we get to new types of motion, but the particle model will serve us well throughout Part I of this book.

STOP TO THINK 1.2 Three motion diagrams are shown. Which is a dust particle settling to the floor at constant speed, which is a ball dropped from the roof of a building, and which is a descending rocket slowing to make a soft landing on Mars?



1.3 Position, Time, and Displacement

To use a motion diagram, you would like to know *where* the object is (i.e., its *position*) and *when* the object was at that position (i.e., the *time*). Position measurements can be made by laying a coordinate-system grid over a motion diagram. You can then measure the (x, y) coordinates of each point in the motion diagram. Of course, the world does not come with a coordinate system attached. A coordinate system is an artificial grid that *you* place over a problem in order to analyze the motion. You place the origin of your coordinate system wherever you wish, and different observers of a moving object might all choose to use different origins.

Time, in a sense, is also a coordinate system, although you may never have thought of time this way. You can pick an arbitrary point in the motion and label it "$t = 0$ seconds." This is simply the instant you decide to start your clock or stopwatch, so it is the origin of your time coordinate. Different observers might choose to start their clocks at different moments. A video frame labeled "$t = 4$ seconds" was taken 4 seconds after you started your clock.

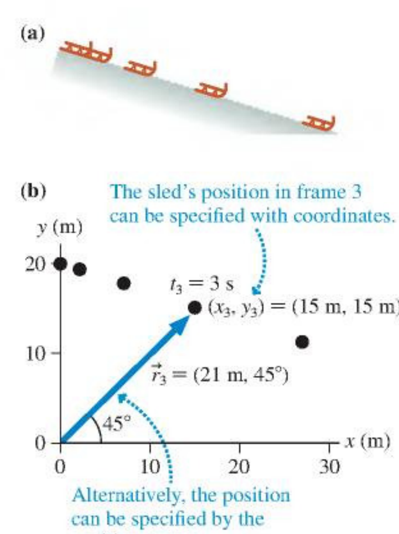
We typically choose $t = 0$ to represent the "beginning" of a problem, but the object may have been moving before then. Those earlier instants would be measured as negative times, just as objects on the x-axis to the left of the origin have negative values of position. Negative numbers are not to be avoided; they simply locate an event in space or time *relative to an origin*.

To illustrate, **FIGURE 1.5a** shows a sled sliding down a snow-covered hill. **FIGURE 1.5b** is a motion diagram for the sled, over which we've drawn an xy-coordinate system. You can see that the sled's position is $(x_3, y_3) = (15\text{ m}, 15\text{ m})$ at time $t_3 = 3$ s. Notice how we've used subscripts to indicate the time and the object's position in a specific frame of the motion diagram.

NOTE The frame at $t = 0$ is frame 0. That is why the fourth frame is labeled 3.

Another way to locate the sled is to draw its **position vector:** an arrow from the origin to the point representing the sled. The position vector is given the symbol $\vec{r}$. Figure 1.5b shows the position vector $\vec{r}_3 = (21\text{ m}, 45°)$. The position vector $\vec{r}$ does not tell us anything different than the coordinates (x, y). It simply provides the information in an alternative form.

FIGURE 1.5 Motion diagram of a sled with frames made every 1 s.



Scalars and Vectors

Some physical quantities, such as time, mass, and temperature, can be described completely by a single number with a unit. For example, the mass of an object is 6 kg and its temperature is 30°C. A single number (with a unit) that describes a physical quantity is called a **scalar.** A scalar can be positive, negative, or zero.

Many other quantities, however, have a directional aspect and cannot be described by a single number. To describe the motion of a car, for example, you must specify not only how fast it is moving, but also the *direction* in which it is moving. A quantity having both a *size* (the "How far?" or "How fast?") and a *direction* (the "Which way?") is called a **vector.** The size or length of a vector is called its *magnitude*. Vectors will be studied thoroughly in Chapter 3, so all we need for now is a little basic information.

We indicate a vector by drawing an arrow over the letter that represents the quantity. Thus $\vec{r}$ and $\vec{A}$ are symbols for vectors, whereas r and A, without the arrows, are symbols for scalars. In handwritten work you must draw arrows over all symbols that represent vectors. This may seem strange until you get used to it, but it is very important because we will often use both r and $\vec{r}$, or both A and $\vec{A}$, in the same problem, and they mean different things! Note that the arrow over the symbol always points to the right, regardless of which direction the actual vector points. Thus we write $\vec{r}$ or $\vec{A}$, never $\overleftarrow{r}$ or $\overleftarrow{A}$.

Displacement

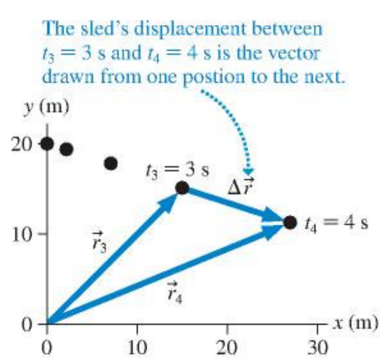


FIGURE 1.6 The sled undergoes a displacement $\Delta\vec{r}$ from position $\vec{r}_3$ to position $\vec{r}_4$.

We said that motion is the change in an object's position with time, but how do we show a change of position? A motion diagram is the perfect tool. **FIGURE 1.6** is the motion diagram of a sled sliding down a snow-covered hill. To show how the sled's position changes between, say, $t_3 = 3$ s and $t_4 = 4$ s, we draw a vector arrow between the two dots of the motion diagram. This vector is the sled's **displacement,** which is given the symbol $\Delta\vec{r}$. The Greek letter delta (Δ) is used in math and science to indicate the *change* in a quantity. In this case, as we'll show, the displacement $\Delta\vec{r}$ is the change in an object's position.

NOTE $\Delta\vec{r}$ is a *single* symbol. You cannot cancel out or remove the Δ.

Notice how the sled's position vector $\vec{r}_4$ is a combination of its early position $\vec{r}_3$ with the displacement vector $\Delta\vec{r}$. In fact, $\vec{r}_4$ is the *vector sum* of the vectors $\vec{r}_3$ and $\Delta\vec{r}$. This is written

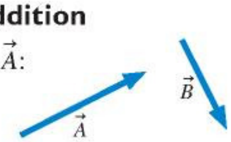
$$\vec{r}_4 = \vec{r}_3 + \Delta\vec{r} \tag{1.1}$$

Here we're adding vector quantities, not numbers, and vector addition differs from "regular" addition. We'll explore vector addition more thoroughly in Chapter 3, but for now you can add two vectors $\vec{A}$ and $\vec{B}$ with the three-step procedure shown in Tactics Box 1.1.

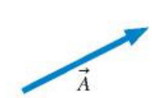
TACTICS BOX 1.1 MP

Vector addition

To add $\vec{B}$ to $\vec{A}$:

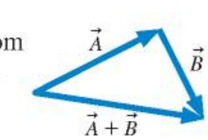


❶ Draw $\vec{A}$.



❷ Place the tail of $\vec{B}$ at the tip of $\vec{A}$.

❸ Draw an arrow from the tail of $\vec{A}$ to the tip of $\vec{B}$. This is vector $\vec{A} + \vec{B}$.



If you examine Figure 1.6, you'll see that the steps of Tactics Box 1.1 are exactly how $\vec{r}_3$ and $\Delta\vec{r}$ are added to give $\vec{r}_4$.

NOTE A vector is not tied to a particular location on the page. You can move a vector around as long as you don't change its length or the direction it points. Vector $\vec{B}$ is not changed by sliding it to where its tail is at the tip of $\vec{A}$.

Equation 1.1 told us that $\vec{r}_4 = \vec{r}_3 + \Delta\vec{r}$. This is easily rearranged to give a more precise definition of displacement: **The displacement $\Delta\vec{r}$ of an object as it moves from an initial position $\vec{r}_i$ to a final position $\vec{r}_f$ is**

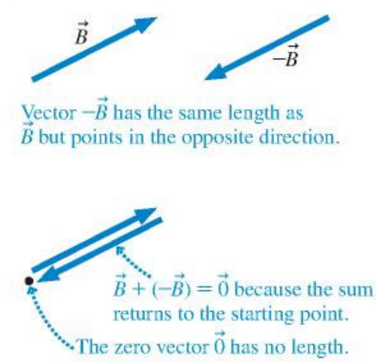
$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i \tag{1.2}$$

Graphically, $\Delta\vec{r}$ is a vector arrow drawn from position $\vec{r}_i$ to position $\vec{r}_f$.

NOTE To be more general, we've written Equation 1.2 in terms of an *initial position* and a *final position,* indicated by subscripts i and f. We'll frequently use i and f when writing general equations, then use specific numbers or values, such as 3 and 4, when working a problem.

This definition of $\Delta\vec{r}$ involves *vector subtraction.* With numbers, subtraction is the same as the addition of a negative number. That is, $5 - 3$ is the same as $5 + (-3)$. Similarly, we can use the rules for vector addition to find $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ if we first define what we mean by $-\vec{B}$. As FIGURE 1.7 shows, the negative of vector $\vec{B}$ is a vector with the same length but pointing in the opposite direction. This makes sense because $\vec{B} - \vec{B} = \vec{B} + (-\vec{B}) = \vec{0}$, where $\vec{0}$, a vector with zero length, is called the **zero vector.**

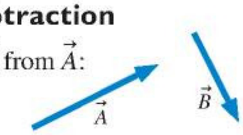
FIGURE 1.7 The negative of a vector.



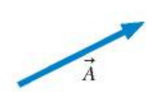
TACTICS BOX 1.2

Vector subtraction

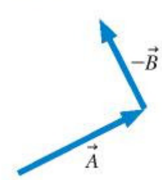
To subtract $\vec{B}$ from $\vec{A}$:



1. Draw $\vec{A}$.



2. Place the tail of $-\vec{B}$ at the tip of $\vec{A}$.



3. Draw an arrow from the tail of $\vec{A}$ to the tip of $-\vec{B}$. This is vector $\vec{A} - \vec{B}$.

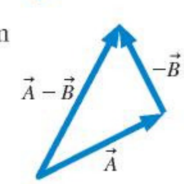


FIGURE 1.8 uses the vector subtraction rules of Tactics Box 1.2 to prove that the displacement $\Delta\vec{r}$ is simply the vector connecting the dots of a motion diagram.

▼ FIGURE 1.8 Using vector subtraction to find $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$.

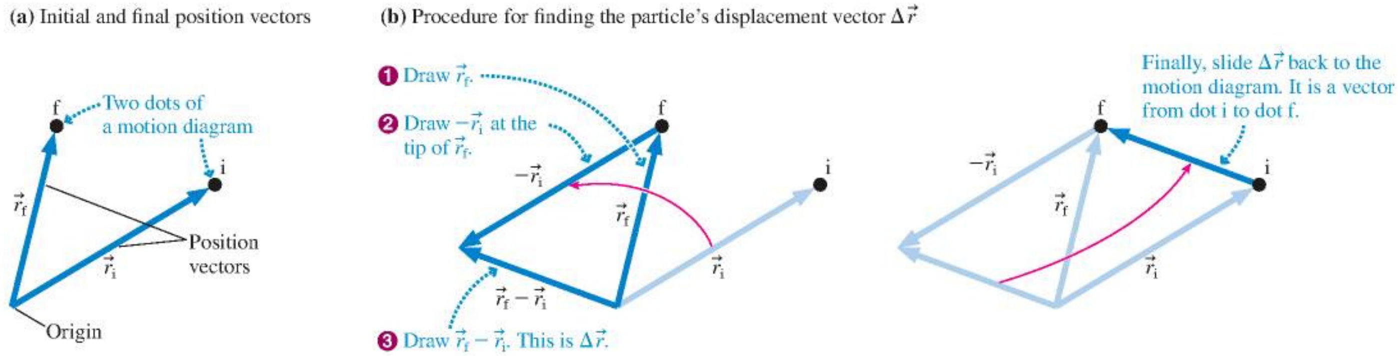
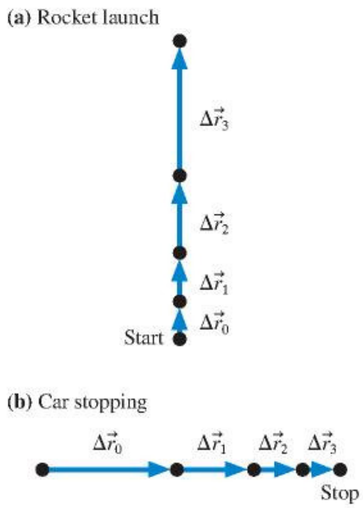


FIGURE 1.9 Motion diagrams with the displacement vectors.



Motion Diagrams with Displacement Vectors

The first step in analyzing a motion diagram is to determine all of the displacement vectors. As Figure 1.8 shows, the displacement vectors are simply the arrows connecting each dot to the next. Label each arrow with a *vector* symbol $\Delta\vec{r}_n$, starting with $n = 0$. FIGURE 1.9 shows the motion diagrams of Figure 1.4 redrawn to include the displacement vectors. You do not need to show the position vectors.

NOTE When an object either starts from rest or ends at rest, the initial or final dots are *as close together* as you can draw the displacement vector arrow connecting them. In addition, just to be clear, you should write "Start" or "Stop" beside the initial or final dot. It is important to distinguish stopping from merely slowing down.

Now we can conclude, more precisely than before, that, as time proceeds:

- An object is speeding up if its displacement vectors are increasing in length.
- An object is slowing down if its displacement vectors are decreasing in length.

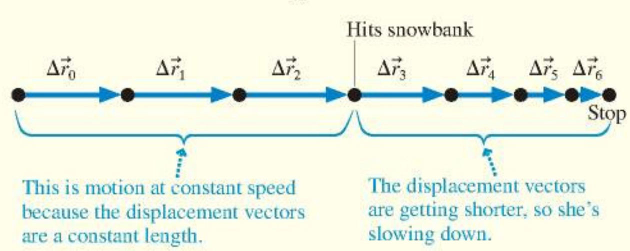
EXAMPLE 1.1 Headfirst into the snow

Alice is sliding along a smooth, icy road on her sled when she suddenly runs headfirst into a large, very soft snowbank that gradually brings her to a halt. Draw a motion diagram for Alice. Show and label all displacement vectors.

MODEL The details of Alice and the sled—their size, shape, color, and so on—are not relevant to understanding their overall motion. So we can model Alice and the sled as one particle.

VISUALIZE FIGURE 1.10 shows a motion diagram. The problem statement suggests that the sled's speed is very nearly constant until it hits the snowbank. Thus the displacement vectors are of equal length as Alice slides along the icy road. She begins slowing when she hits the snowbank, so the displacement vectors then get shorter until the sled stops. We're told that her stop is gradual, so we want the vector lengths to get shorter gradually rather than suddenly.

FIGURE 1.10 The motion diagram of Alice and the sled.



Time Interval

A stopwatch is used to measure a time interval.

It's also useful to consider a *change* in time. For example, the clock readings of two frames of film might be t_1 and t_2. The specific values are arbitrary because they are timed relative to an arbitrary instant that you chose to call $t = 0$. But the **time interval** $\Delta t = t_2 - t_1$ is *not* arbitrary. It represents the elapsed time for the object to move from one position to the next.

The time interval $\Delta t = t_f - t_i$ measures the elapsed time as an object moves from an initial position $\vec{r}_i$ at time t_i to a final position $\vec{r}_f$ at time t_f. The value of Δt is independent of the specific clock used to measure the times.

To summarize the main idea of this section, we have added coordinate systems and clocks to our motion diagrams in order to measure *when* each frame was exposed and *where* the object was located at that time. Different observers of the motion may choose different coordinate systems and different clocks. However, all observers find the *same* values for the displacements $\Delta\vec{r}$ and the time intervals Δt because these are independent of the specific coordinate system used to measure them.

1.4 Velocity

It's no surprise that, during a given time interval, a speeding bullet travels farther than a speeding snail. To extend our study of motion so that we can compare the bullet to the snail, we need a way to measure how fast or how slowly an object moves.

One quantity that measures an object's fastness or slowness is its **average speed,** defined as the ratio

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval spent traveling}} = \frac{d}{\Delta t} \tag{1.3}$$

If you drive 15 miles (mi) in 30 minutes $(\frac{1}{2}\text{ h})$, your average speed is

$$\text{average speed} = \frac{15 \text{ mi}}{\frac{1}{2}\text{ h}} = 30 \text{ mph} \tag{1.4}$$

The victory goes to the runner with the highest average speed.

Although the concept of speed is widely used in our day-to-day lives, it is not a sufficient basis for a science of motion. To see why, imagine you're trying to land a jet plane on an aircraft carrier. It matters a great deal to you whether the aircraft carrier is moving at 20 mph (miles per hour) to the north or 20 mph to the east. Simply knowing that the boat's speed is 20 mph is not enough information!

It's the displacement $\Delta\vec{r}$, a vector quantity, that tells us not only the distance traveled by a moving object, but also the *direction* of motion. Consequently, a more useful ratio than $d/\Delta t$ is the ratio $\Delta\vec{r}/\Delta t$. In addition to measuring how fast an object moves, this ratio is a vector that points in the direction of motion.

It is convenient to give this ratio a name. We call it the **average velocity,** and it has the symbol $\vec{v}_{\text{avg}}$. **The average velocity of an object during the time interval Δt, in which the object undergoes a displacement $\Delta\vec{r}$, is the vector**

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \tag{1.5}$$

An object's average velocity vector points in the same direction as the displacement vector $\Delta\vec{r}$. This is the direction of motion.

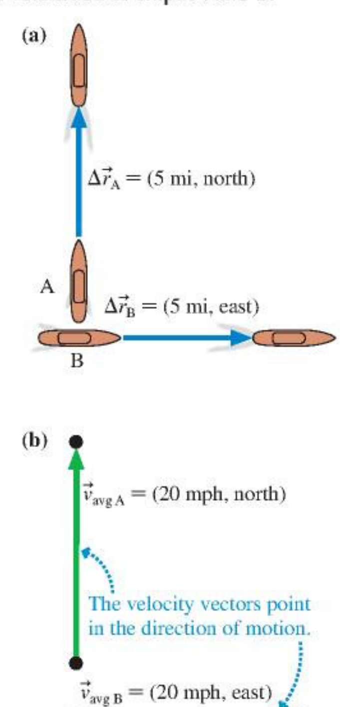
NOTE In everyday language we do not make a distinction between speed and velocity, but in physics *the distinction is very important.* In particular, speed is simply "How fast?" whereas velocity is "How fast, and in which direction?" As we go along we will be giving other words more precise meanings in physics than they have in everyday language.

As an example, **FIGURE 1.11a** shows two ships that move 5 miles in 15 minutes. Using Equation 1.5 with $\Delta t = 0.25$ h, we find

$$\begin{aligned} \vec{v}_{\text{avg A}} &= (20 \text{ mph, north}) \\ \vec{v}_{\text{avg B}} &= (20 \text{ mph, east}) \end{aligned} \tag{1.6}$$

Both ships have a speed of 20 mph, but their velocities are different. Notice how the velocity *vectors* in **FIGURE 1.11b** point in the direction of motion.

FIGURE 1.11 The displacement vectors and velocities of ships A and B.



NOTE Our goal in this chapter is to *visualize* motion with motion diagrams. Strictly speaking, the vector we have defined in Equation 1.5, and the vector we will show on motion diagrams, is the *average* velocity $\vec{v}_{\text{avg}}$. But to allow the motion diagram to be a useful tool, we will drop the subscript and refer to the average velocity as simply $\vec{v}$. Our definitions and symbols, which somewhat blur the distinction between average and instantaneous quantities, are adequate for visualization purposes, but they're not the final word. We will refine these definitions in Chapter 2, where our goal will be to develop the mathematics of motion.

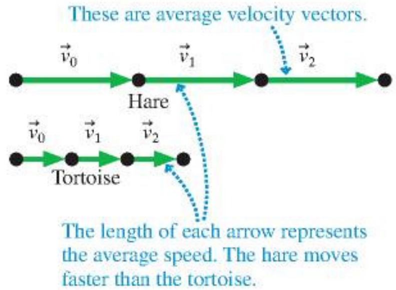
Motion Diagrams with Velocity Vectors

The velocity vector points in the same direction as the displacement $\Delta\vec{r}$, and the length of $\vec{v}$ is directly proportional to the length of $\Delta\vec{r}$. Consequently, the vectors connecting each dot of a motion diagram to the next, which we previously labeled as displacements, could equally well be identified as velocity vectors.

This idea is illustrated in FIGURE 1.12, which shows four frames from the motion diagram of a tortoise racing a hare. The vectors connecting the dots are now labeled as velocity vectors $\vec{v}$. **The length of a velocity vector represents the average speed with which the object moves between the two points.** Longer velocity vectors indicate faster motion. You can see that the hare moves faster than the tortoise.

Notice that the hare's velocity vectors do not change; each has the same length and direction. We say the hare is moving with *constant velocity*. The tortoise is also moving with its own constant velocity.

FIGURE 1.12 Motion diagram of the tortoise racing the hare.



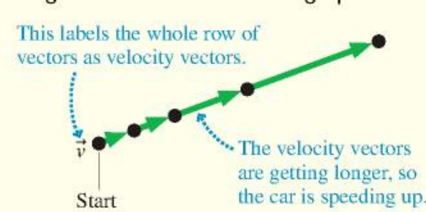
EXAMPLE 1.2 Accelerating up a hill

The light turns green and a car accelerates, starting from rest, up a 20° hill. Draw a motion diagram showing the car's velocity.

MODEL Use the particle model to represent the car as a dot.

VISUALIZE The car's motion takes place along a straight line, but the line is neither horizontal nor vertical. A motion diagram should show the object moving with the correct orientation—in this case, at an angle of 20°. FIGURE 1.13 shows several frames of the motion diagram, where we see the car speeding up. The car starts from rest, so the first arrow is drawn as short as possible and the first dot is labeled "Start." The displacement vectors have been drawn from each dot to the next, but then they are identified and labeled as average velocity vectors $\vec{v}$.

FIGURE 1.13 Motion diagram of a car accelerating up a hill.



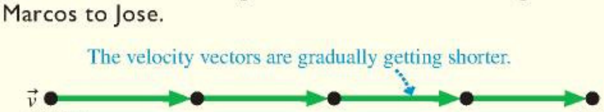
EXAMPLE 1.3 A rolling soccer ball

Marcos kicks a soccer ball. It rolls along the ground until stopped by Jose. Draw a motion diagram of the ball.

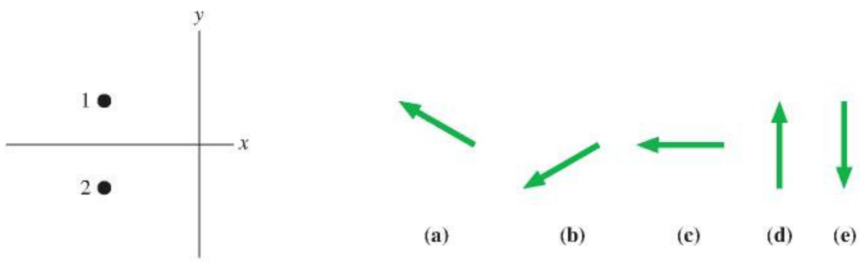
MODEL This example is typical of how many problems in science and engineering are worded. The problem does not give a clear statement of where the motion begins or ends. Are we interested in the motion of the ball just during the time it is rolling between Marcos and Jose? What about the motion *as* Marcos kicks it (ball rapidly speeding up) or *as* Jose stops it (ball rapidly slowing down)? The point is that *you* will often be called on to make a *reasonable interpretation* of a problem statement. In this problem, the details of kicking and stopping the ball are complex. The motion of the ball across the ground is easier to describe, and it's a motion you might expect to learn about in a physics class. So our *interpretation* is that the motion diagram should start as the ball leaves Marcos's foot (ball already moving) and should end the instant it touches Jose's foot (ball still moving). In between, the ball will slow down a little. We will model the ball as a particle.

VISUALIZE With this interpretation in mind, FIGURE 1.14 shows the motion diagram of the ball. Notice how, in contrast to the car of Figure 1.13, the ball is already moving as the motion diagram video begins. As before, the average velocity vectors are found by connecting the dots. You can see that the average velocity vectors get shorter as the ball slows. Each $\vec{v}$ is different, so this is *not* constant-velocity motion.

FIGURE 1.14 Motion diagram of a soccer ball rolling from Marcos to Jose.



STOP TO THINK 1.3 A particle moves from position 1 to position 2 during the time interval Δt. Which vector shows the particle's average velocity?



1.5 Linear Acceleration

Position, time, and velocity are important concepts, and at first glance they might appear to be sufficient to describe motion. But that is not the case. Sometimes an object's velocity is constant, as it was in Figure 1.12. More often, an object's velocity changes as it moves, as in Figures 1.13 and 1.14. We need one more motion concept to describe a *change* in the velocity.

Because velocity is a vector, it can change in two possible ways:

1. The magnitude can change, indicating a change in speed; or
2. The direction can change, indicating that the object has changed direction.

We will concentrate for now on the first case, a change in speed. The car accelerating up a hill in Figure 1.13 was an example in which the magnitude of the velocity vector changed but not the direction. We'll return to the second case in Chapter 4.

When we wanted to measure changes in position, the ratio $\Delta\vec{r}/\Delta t$ was useful. This ratio is the *rate of change of position*. By analogy, consider an object whose velocity changes from an initial $\vec{v}_i$ to a final $\vec{v}_f$ during the time interval Δt. Just as $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$ is the change of position, the quantity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ is the change of velocity. The ratio $\Delta\vec{v}/\Delta t$ is then the *rate of change of velocity*. It has a large magnitude for objects that speed up quickly and a small magnitude for objects that speed up slowly.

The ratio $\Delta\vec{v}/\Delta t$ is called the **average acceleration,** and its symbol is $\vec{a}_{avg}$. **The average acceleration of an object during the time interval Δt, in which the object's velocity changes by $\Delta\vec{v}$, is the vector**

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \tag{1.7}$$

The average acceleration vector points in the same direction as the vector $\Delta\vec{v}$.

Acceleration is a fairly abstract concept. Yet it is essential to develop a good intuition about acceleration because it will be a key concept for understanding why objects move as they do. Motion diagrams will be an important tool for developing that intuition.

The Audi TT accelerates from 0 to 60 mph in 6 s.

NOTE As we did with velocity, we will drop the subscript and refer to the average acceleration as simply $\vec{a}$. This is adequate for visualization purposes, but not the final word. We will refine the definition of acceleration in Chapter 2.

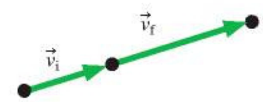
Finding the Acceleration Vectors on a Motion Diagram

Let's look at how we can determine the average acceleration vector $\vec{a}$ from a motion diagram. From its definition, Equation 1.7, we see that **$\vec{a}$ points in the same direction as $\Delta\vec{v}$,** the change of velocity. This critical idea is the basis for a technique to find $\vec{a}$.

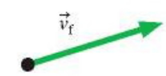
TACTICS BOX 1.3 MP

Finding the acceleration vector

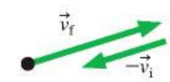
To find the acceleration as the velocity changes from $\vec{v}_i$ to $\vec{v}_f$, we must determine the *change* of velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$.



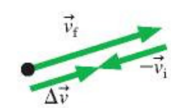
❶ Draw the velocity vector $\vec{v}_f$.



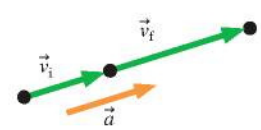
❷ Draw $-\vec{v}_i$ at the tip of $\vec{v}_f$.



❸ Draw $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$
$= \vec{v}_f + (-\vec{v}_i)$
This is the direction of $\vec{a}$.



❹ Return to the original motion diagram. Draw a vector at the middle dot in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_i$ and $\vec{v}_f$.



Exercises 21–24

Many Tactics Boxes will refer you to exercises in the *Student Workbook* where you can practice the new skill.

Notice that the acceleration vector goes beside the middle dot, not beside the velocity vectors. This is because each acceleration vector is determined by the *difference* between the *two* velocity vectors on either side of a dot. The length of $\vec{a}$ does not have to be the exact length of $\Delta\vec{v}$; it is the direction of $\vec{a}$ that is most important.

The procedure of Tactics Box 1.3 can be repeated to find $\vec{a}$ at each point in the motion diagram. Note that we cannot determine $\vec{a}$ at the first and last points because we have only one velocity vector and can't find $\Delta\vec{v}$.

The Complete Motion Diagram

You've now seen several *Tactics Boxes* that help you accomplish specific tasks. Tactics Boxes will appear in nearly every chapter in this book. We'll also, where appropriate, provide *Problem-Solving Strategies*.

PROBLEM-SOLVING STRATEGY 1.1

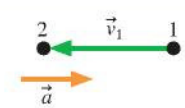
Motion diagrams

MODEL Determine whether it is appropriate to model the moving object as a particle. Make simplifying assumptions when interpreting the problem statement.

VISUALIZE A complete motion diagram consists of:

- The position of the object in each frame of the video, shown as a dot. Use five or six dots to make the motion clear but without overcrowding the picture. More complex motions may need more dots.
- The average velocity vectors, found by connecting each dot in the motion diagram to the next with a vector arrow. There is *one* velocity vector linking each *two* position dots. Label the row of velocity vectors $\vec{v}$.
- The average acceleration vectors, found using Tactics Box 1.3. There is *one* acceleration vector linking each *two* velocity vectors. Each acceleration vector is drawn at the dot between the two velocity vectors it links. Use $\vec{0}$ to indicate a point at which the acceleration is zero. Label the row of acceleration vectors $\vec{a}$.

STOP TO THINK 1.4 A particle undergoes acceleration $\vec{a}$ while moving from point 1 to point 2. Which of the choices shows the most likely velocity vector $\vec{v}_2$ as the particle leaves point 2?



Examples of Motion Diagrams

Let's look at some examples of the full strategy for drawing motion diagrams.

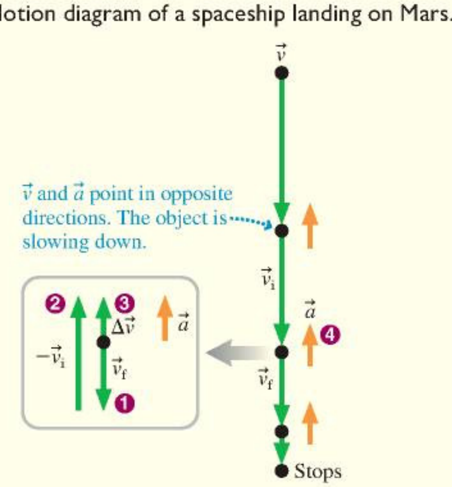
EXAMPLE 1.4 **The first astronauts land on Mars**

A spaceship carrying the first astronauts to Mars descends safely to the surface. Draw a motion diagram for the last few seconds of the descent.

MODEL The spaceship is small in comparison with the distance traveled, and the spaceship does not change size or shape, so it's reasonable to model the spaceship as a particle. We'll assume that its motion in the last few seconds is straight down. The problem ends as the spacecraft touches the surface.

VISUALIZE FIGURE 1.15 shows a complete motion diagram as the spaceship descends and slows, using its rockets, until it comes to rest on the surface. Notice how the dots get closer together as it slows. The inset uses the steps of Tactics Box 1.3 (numbered circles) to show how the acceleration vector $\vec{a}$ is determined at one point. All the other acceleration vectors will be similar because for each pair of velocity vectors the earlier one is longer than the later one.

FIGURE 1.15 Motion diagram of a spaceship landing on Mars.



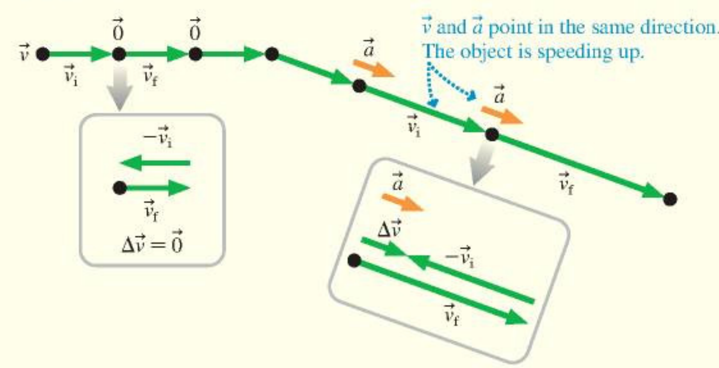
EXAMPLE 1.5 Skiing through the woods

A skier glides along smooth, horizontal snow at constant speed, then speeds up going down a hill. Draw the skier' motion diagram.

MODEL Model the skier as a particle. It's reasonable to assume that the downhill slope is a straight line. Although the motion as a whole is not linear, we can treat the skier's motion as two separate linear motions.

VISUALIZE FIGURE 1.16 shows a complete motion diagram of the skier. The dots are equally spaced for the horizontal motion, indicating constant speed; then the dots get farther apart as the skier speeds up going down the hill. The insets show how the average acceleration vector $\vec{a}$ is determined for the horizontal motion and along the slope. All the other acceleration vectors along the slope will be similar to the one shown because each velocity vector is longer than the preceding one. Notice that we've explicitly written $\vec{0}$ for the acceleration beside the dots where the velocity is constant. The acceleration at the point where the direction changes will be considered in Chapter 4.

FIGURE 1.16 Motion diagram of a skier.



Notice something interesting in Figures 1.15 and 1.16. Where the object is speeding up, the acceleration and velocity vectors point in the *same direction*. Where the object is slowing down, the acceleration and velocity vectors point in *opposite directions*. These results are always true for motion in a straight line. **For motion along a line:**

- **An object is speeding up if and only if $\vec{v}$ and $\vec{a}$ point in the same direction.**
- **An object is slowing down if and only if $\vec{v}$ and $\vec{a}$ point in opposite directions.**
- **An object's velocity is constant if and only if $\vec{a} = \vec{0}$.**

NOTE In everyday language, we use the word *accelerate* to mean "speed up" and the word *decelerate* to mean "slow down." But speeding up and slowing down are both changes in the velocity and consequently, by our definition, *both* are accelerations. In physics, *acceleration* refers to changing the velocity, no matter what the change is, and not just to speeding up.

EXAMPLE 1.6 Tossing a ball

Draw the motion diagram of a ball tossed straight up in the air.

MODEL This problem calls for some interpretation. Should we include the toss itself, or only the motion after the ball is released? Should we include the ball hitting the ground? It appears that this problem is really concerned with the ball's motion through the air. Consequently, we begin the motion diagram at the instant that the tosser releases the ball and end the diagram at the instant the ball hits the ground. We will consider neither the toss nor the impact. And, of course, we will model the ball as a particle.

VISUALIZE We have a slight difficulty here because the ball retraces its route as it falls. A literal motion diagram would show the upward motion and downward motion on top of each other, leading to confusion. We can avoid this difficulty by horizontally separating the upward motion and downward motion diagrams. This will not affect our conclusions because it does not change any of the vectors. FIGURE 1.17 shows the motion diagram drawn this way. Notice that the very top dot is shown twice—as the end point of the upward motion and the beginning point of the downward motion.

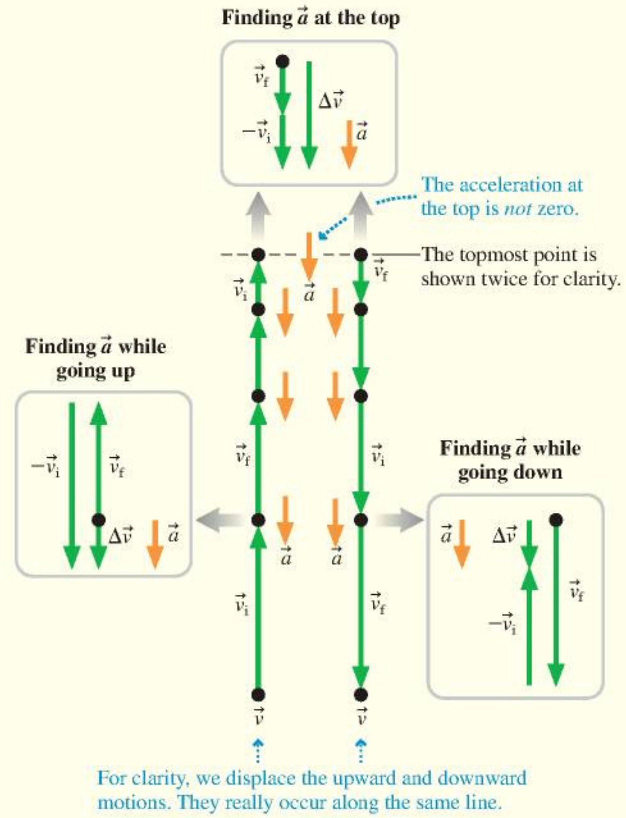
The ball slows down as it rises. You've learned that the acceleration vectors point opposite the velocity vectors for an object that is slowing down along a line, and they are shown accordingly. Similarly, $\vec{a}$ and $\vec{v}$ point in the same direction as the falling ball speeds up. Notice something interesting: The acceleration vectors point downward both while the ball is rising *and* while it is falling. Both "speeding up" and "slowing down" occur with the *same* acceleration vector. This is an important conclusion, one worth pausing to think about.

Now let's look at the top point on the ball's trajectory. The velocity vectors point upward but are getting shorter as the ball approaches the top. As the ball starts to fall, the velocity vectors point downward and are getting longer. There must be a moment—just an instant as $\vec{v}$ switches from pointing up to pointing down—when the velocity is zero. Indeed, the ball's velocity *is* zero for an instant at the precise top of the motion!

But what about the acceleration at the top? The inset shows how the average acceleration is determined from the last upward velocity before the top point and the first downward velocity. We find that the acceleration at the top is pointing downward, just as it does elsewhere in the motion.

Many people expect the acceleration to be zero at the highest point. But the velocity at the top point *is* changing—from up to down. If the velocity is changing, there *must* be an acceleration. A downward-pointing acceleration vector is needed to turn the velocity vector from up to down. Another way to think about this is to note that zero acceleration would mean no change of velocity. When the ball reached zero velocity at the top, it would hang there and not fall if the acceleration were also zero!

FIGURE 1.17 Motion diagram of a ball tossed straight up in the air.



1.6 Motion in One Dimension

An object's motion can be described in terms of three fundamental quantities: its position $\vec{r}$, velocity $\vec{v}$, and acceleration $\vec{a}$. These are vectors, but for motion in one dimension, the vectors are restricted to point only "forward" or "backward." Consequently, we can describe one-dimensional motion with the simpler quantities x, v_x, and a_x (or y, v_y, and a_y). However, we need to give each of these quantities an explicit *sign,* positive or negative, to indicate whether the position, velocity, or acceleration vector points forward or backward.

Determining the Signs of Position, Velocity, and Acceleration

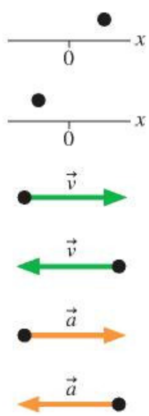
Position, velocity, and acceleration are measured with respect to a coordinate system, a grid or axis that *you* impose on a problem to analyze the motion. We will find it convenient to use an x-axis to describe both horizontal motion and motion along an inclined plane. A y-axis will be used for vertical motion. A coordinate axis has two essential features:

1. An origin to define zero; and
2. An x or y label (with units) to indicate the positive end of the axis.

In this textbook, we will follow the convention that **the positive end of an x-axis is to the right and the positive end of a y-axis is up.** The signs of position, velocity, and acceleration are based on this convention.

TACTICS BOX 1.4

Determining the sign of the position, velocity, and acceleration



$x > 0$ Position to right of origin.

$x < 0$ Position to left of origin.

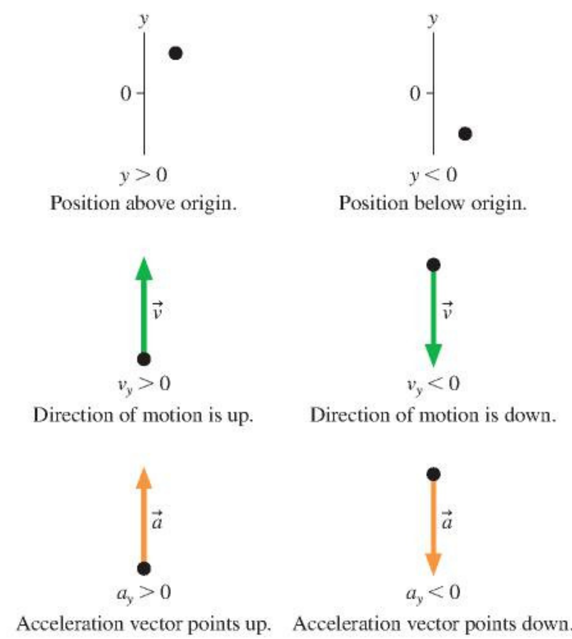
$v_x > 0$ Direction of motion is to the right.

$v_x < 0$ Direction of motion is to the left.

$a_x > 0$ Acceleration vector points to the right.

$a_x < 0$ Acceleration vector points to the left.

- The sign of position (x or y) tells us *where* an object is.
- The sign of velocity (v_x or v_y) tells us *which direction* the object is moving.
- The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, *not* whether the object is speeding up or slowing down.



Exercises 30–31

Acceleration is where things get a bit tricky. A natural tendency is to think that a positive value of a_x or a_y describes an object that is speeding up while a negative value describes an object that is slowing down (decelerating). However, this interpretation *does not work.*

Acceleration is defined as $\vec{a}_{avg} = \Delta\vec{v}/\Delta t$. The direction of $\vec{a}$ can be determined by using a motion diagram to find the direction of $\Delta\vec{v}$. The one-dimensional acceleration a_x (or a_y) is then positive if the vector $\vec{a}$ points to the right (or up), negative if $\vec{a}$ points to the left (or down).

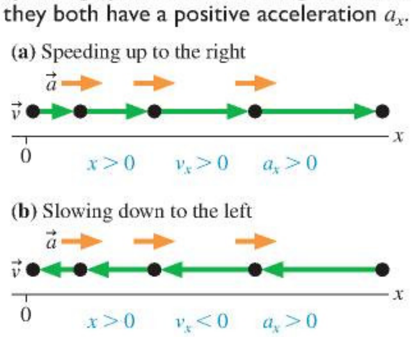
FIGURE 1.18 shows that this method for determining the sign of a does not conform to the simple idea of speeding up and slowing down. The object in Figure 1.18a has a positive acceleration ($a_x > 0$) not because it is speeding up but because the vector $\vec{a}$ points in the positive direction. Compare this with the motion diagram of Figure 1.18b. Here the object is slowing down, but it still has a positive acceleration ($a_x > 0$) because $\vec{a}$ points to the right.

In the previous section, we found that an object is speeding up if $\vec{v}$ and $\vec{a}$ point in the same direction, slowing down if they point in opposite directions. For one-dimensional motion this rule becomes:

- An object is speeding up if and only if v_x and a_x have the same sign.
- An object is slowing down if and only if v_x and a_x have opposite signs.
- An object's velocity is constant if and only if $a_x = 0$.

Notice how the first two of these rules are at work in Figure 1.18.

FIGURE 1.18 One of these objects is speeding up, the other slowing down, but they both have a positive acceleration a_x.



Position-versus-Time Graphs

FIGURE 1.19 is a motion diagram, made at 1 frame per minute, of a student walking to school. You can see that she leaves home at a time we choose to call $t = 0$ min and makes steady progress for a while. Beginning at $t = 3$ min there is a period where the distance traveled during each time interval becomes less—perhaps she slowed down to speak with a friend. Then she picks up the pace, and the distances within each interval are longer.

FIGURE 1.19 The motion diagram of a student walking to school and a coordinate axis for making measurements.

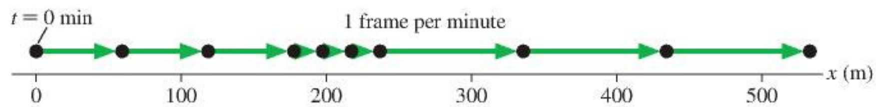


Figure 1.19 includes a coordinate axis, and you can see that every dot in a motion diagram occurs at a specific position. TABLE 1.1 shows the student's positions at different times as measured along this axis. For example, she is at position $x = 120$ m at $t = 2$ min.

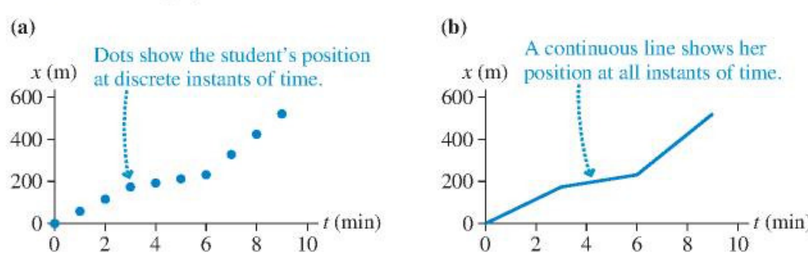
The motion diagram is one way to represent the student's motion. Another is to make a graph of the measurements in Table 1.1. FIGURE 1.20a is a graph of x versus t for the student. The motion diagram tells us only where the student is at a few discrete points of time, so this graph of the data shows only points, no lines.

NOTE A graph of "*a* versus *b*" means that *a* is graphed on the vertical axis and *b* on the horizontal axis. Saying "graph *a* versus *b*" is really a shorthand way of saying "graph *a* as a function of *b*."

TABLE 1.1 Measured positions of a student walking to school

Time t (min)	Position x (m)	Time t (min)	Position x (m)
0	0	5	220
1	60	6	240
2	120	7	340
3	180	8	440
4	200	9	540

FIGURE 1.20 Position graphs of the student's motion.



However, common sense tells us the following. First, the student was *somewhere specific* at all times. That is, there was never a time when she failed to have a well-defined position, nor could she occupy two positions at one time. (As reasonable as this belief appears to be, it will be severely questioned and found not entirely accurate when we get to quantum physics!) Second, the student moved *continuously* through all intervening points of space. She could not go from $x = 100$ m to $x = 200$ m without passing through every point in between. It is thus quite reasonable to believe that her motion can be shown as a continuous line passing through the measured points, as shown in FIGURE 1.20b. A continuous line or curve showing an object's position as a function of time is called a **position-versus-time graph** or, sometimes, just a *position graph*.

NOTE A graph is *not* a "picture" of the motion. The student is walking along a straight line, but the graph itself is not a straight line. Further, we've graphed her position on the vertical axis even though her motion is horizontal. Graphs are *abstract representations* of motion. We will place significant emphasis on the process of interpreting graphs, and many of the exercises and problems will give you a chance to practice these skills.

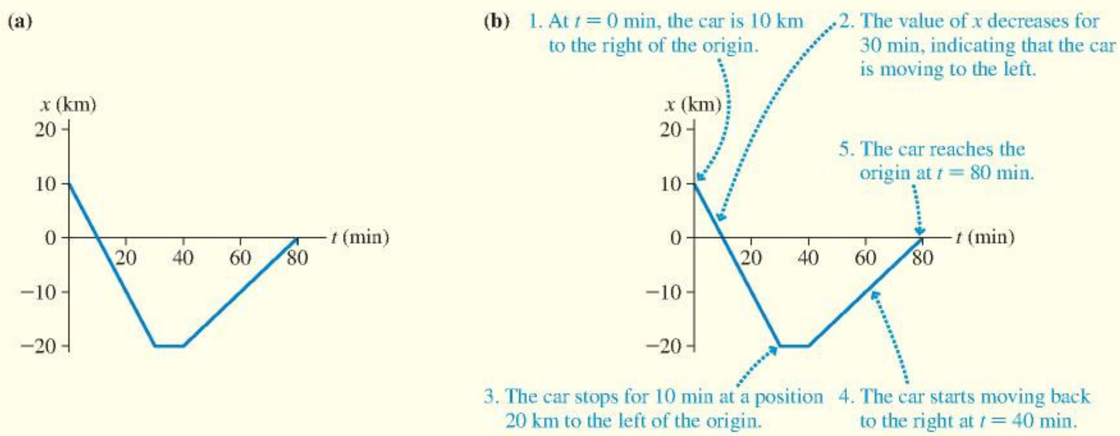
EXAMPLE 1.7 **Interpreting a position graph**

The graph in **FIGURE 1.21a** represents the motion of a car along a straight road. Describe the motion of the car.

MODEL We'll model the car as a particle with a precise position at each instant.

VISUALIZE As **FIGURE 1.21b** shows, the graph represents a car that travels to the left for 30 minutes, stops for 10 minutes, then travels back to the right for 40 minutes.

FIGURE 1.21 Position-versus-time graph of a car.



1.7 Solving Problems in Physics

Physics is not mathematics. Math problems are clearly stated, such as "What is $2 + 2$?" Physics is about the world around us, and to describe that world we must use language. Now, language is wonderful—we couldn't communicate without it—but language can sometimes be imprecise or ambiguous.

The challenge when reading a physics problem is to translate the words into symbols that can be manipulated, calculated, and graphed. **The translation from words to symbols is the heart of problem solving in physics.** This is the point where ambiguous words and phrases must be clarified, where the imprecise must be made precise, and where you arrive at an understanding of exactly what the question is asking.

Using Symbols

Symbols are a language that allows us to talk with precision about the relationships in a problem. As with any language, we all need to agree to use words or symbols in the same way if we want to communicate with each other. Many of the ways we use symbols in science and engineering are somewhat arbitrary, often reflecting historical roots. Nonetheless, practicing scientists and engineers have come to agree on how to use the language of symbols. Learning this language is part of learning physics.

We will use subscripts on symbols, such as x_3, to designate a particular point in the problem. Scientists usually label the starting point of the problem with the subscript "0," not the subscript "1" that you might expect. When using subscripts, make sure that all symbols referring to the same point in the problem have the *same numerical subscript*. To have the same point in a problem characterized by position x_1 but velocity v_{2x} is guaranteed to lead to confusion!

Drawing Pictures

You may have been told that the first step in solving a physics problem is to "draw a picture," but perhaps you didn't know why, or what to draw. The purpose of drawing a picture is to aid you in the words-to-symbols translation. Complex problems have far more information than you can keep in your head at one time. Think of a picture as a "memory extension," helping you organize and keep track of vital information.

Although any picture is better than none, there really is a *method* for drawing pictures that will help you be a better problem solver. It is called the **pictorial representation** of the problem. We'll add other pictorial representations as we go along, but the following procedure is appropriate for motion problems.

TACTICS BOX 1.5

Drawing a pictorial representation

1. **Draw a motion diagram.** The motion diagram develops your intuition for the motion.
2. **Establish a coordinate system.** Select your axes and origin to match the motion. For one-dimensional motion, you want either the x-axis or the y-axis parallel to the motion. The coordinate system determines whether the signs of v and a are positive or negative.
3. **Sketch the situation.** Not just any sketch. Show the object at the *beginning* of the motion, at the *end,* and at any point where the character of the motion changes. Show the object, not just a dot, but very simple drawings are adequate.
4. **Define symbols.** Use the sketch to define symbols representing quantities such as position, velocity, acceleration, and time. *Every* variable used later in the mathematical solution should be defined on the sketch. Some will have known values, others are initially unknown, but all should be given symbolic names.
5. **List known information.** Make a table of the quantities whose values you can determine from the problem statement or that can be found quickly with simple geometry or unit conversions. Some quantities are implied by the problem, rather than explicitly given. Others are determined by your choice of coordinate system.
6. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined as symbols in step 4. Don't list every unknown, only the one or two needed to answer the question.

It's not an overstatement to say that a well-done pictorial representation of the problem will take you halfway to the solution. The following example illustrates how to construct a pictorial representation for a problem that is typical of problems you will see in the next few chapters.

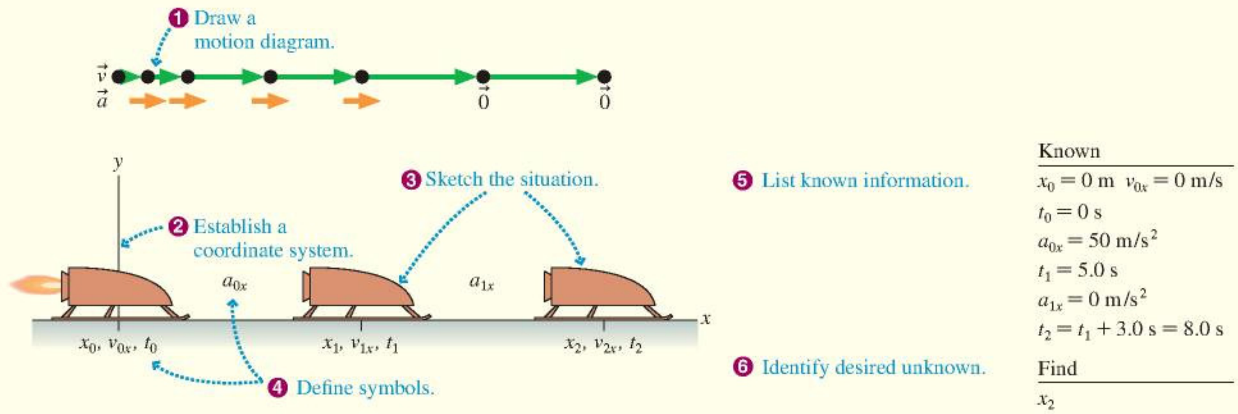
EXAMPLE 1.8 | Drawing a pictorial representation

Draw a pictorial representation for the following problem: A rocket sled accelerates horizontally at 50 m/s^2 for 5.0 s, then coasts for 3.0 s. What is the total distance traveled?

VISUALIZE FIGURE 1.22, on the next page, is the pictorial representation. The motion diagram shows an acceleration phase followed by a coasting phase. Because the motion is horizontal, the appropriate coordinate system is an x-axis. We've chosen to place the origin at the starting point. The motion has a beginning, an end, and a point where the motion changes from accelerating to coasting, and these are the three sled positions sketched in the figure. The quantities x, v_x, and t are needed at each of three *points,* so these have been defined on the sketch and distinguished by subscripts. Accelerations are associated with *intervals* between the points, so only two accelerations are defined. Values for three quantities are given in the problem statement, although we need to use the motion diagram, where $\vec{a}$ points to the right, and our choice of coordinate system to know that $a_{0x} = +50$ m/s^2 rather than -50 m/s^2. The values $x_0 = 0$ m and $t_0 = 0$ s are choices we made when setting up the coordinate system. The value $v_{0x} = 0$ m/s is part of our *interpretation* of the problem. Finally, we identify x_2 as the quantity that will answer the question. We now understand quite a bit about the problem and would be ready to start a quantitative analysis.

Continued

FIGURE 1.22 A pictorial representation.



We didn't *solve* the problem; that is not the purpose of the pictorial representation. The pictorial representation is a systematic way to go about interpreting a problem and getting ready for a mathematical solution. Although this is a simple problem, and you probably know how to solve it if you've taken physics before, you will soon be faced with much more challenging problems. Learning good problem-solving skills at the beginning, while the problems are easy, will make them second nature later when you really need them.

A new building requires careful planning. The architect's visualization and drawings have to be complete before the detailed procedures of construction get under way. The same is true for solving problems in physics.

Representations

A picture is one way to *represent* your knowledge of a situation. You could also represent your knowledge using words, graphs, or equations. Each **representation of knowledge** gives us a different perspective on the problem. The more tools you have for thinking about a complex problem, the more likely you are to solve it.

There are four representations of knowledge that we will use over and over:

1. The *verbal* representation. A problem statement, in words, is a verbal representation of knowledge. So is an explanation that you write.
2. The *pictorial* representation. The pictorial representation, which we've just presented, is the most literal depiction of the situation.
3. The *graphical* representation. We will make extensive use of graphs.
4. The *mathematical* representation. Equations that can be used to find the numerical values of specific quantities are the mathematical representation.

NOTE The mathematical representation is only one of many. Much of physics is more about thinking and reasoning than it is about solving equations.

A Problem-Solving Strategy

One of the goals of this textbook is to help you learn a *strategy* for solving physics problems. The purpose of a strategy is to guide you in the right direction with minimal wasted effort. The four-part problem-solving strategy shown on the next page—**Model, Visualize, Solve, Assess**—is based on using different representations of knowledge. You will see this problem-solving strategy used consistently in the worked examples throughout this textbook, and you should endeavor to apply it to your own problem solving.

Throughout this textbook we will emphasize the first two steps. They are the *physics* of the problem, as opposed to the mathematics of solving the resulting equations. This is not to say that those mathematical operations are always easy—in many cases they are not. But our primary goal is to understand the physics.

GENERAL PROBLEM-SOLVING STRATEGY

MODEL It's impossible to treat every detail of a situation. Simplify the situation with a model that captures the essential features. For example, the object in a mechanics problem is often represented as a particle.

VISUALIZE This is where expert problem solvers put most of their effort.

- Draw a *pictorial representation.* This helps you visualize important aspects of the physics and assess the information you are given. It starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these representations; they need not be done in any particular order.

SOLVE Only after modeling and visualizing are complete is it time to develop a *mathematical representation* with specific equations that must be solved. All symbols used here should have been defined in the pictorial representation.

ASSESS Is your result believable? Does it have proper units? Does it make sense?

Textbook illustrations are obviously more sophisticated than what you would draw on your own paper. To show you a figure very much like what *you* should draw, the final example of this section is in a "pencil sketch" style. We will include one or more pencil-sketch examples in nearly every chapter to illustrate exactly what a good problem solver would draw.

EXAMPLE 1.9 Launching a weather rocket

Use the first two steps of the problem-solving strategy to analyze the following problem: A small rocket, such as those used for meteorological measurements of the atmosphere, is launched vertically with an acceleration of 30 m/s^2. It runs out of fuel after 30 s. What is its maximum altitude?

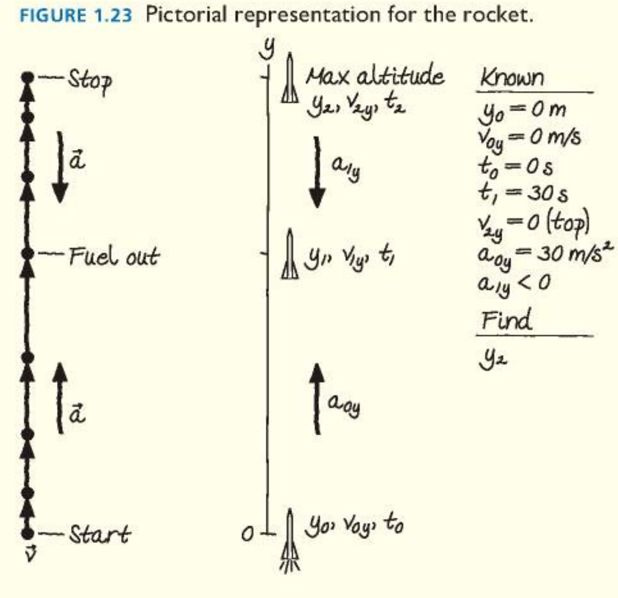
MODEL We need to do some interpretation. Common sense tells us that the rocket does not stop the instant it runs out of fuel. Instead, it continues upward, while slowing, until it reaches its maximum altitude. This second half of the motion, after running out of fuel, is like the ball that was tossed upward in the first half of Example 1.6. Because the problem does not ask about the rocket's descent, we conclude that the problem ends at the point of maximum altitude. We'll model the rocket as a particle.

VISUALIZE FIGURE 1.23 shows the pictorial representation in pencil-sketch style. The rocket is speeding up during the first half of the motion, so $\vec{a}_0$ points upward, in the positive y-direction. Thus the initial acceleration is $a_{0y} = 30$ m/s^2. During the second half, as the rocket slows, $\vec{a}_1$ points downward. Thus a_{1y} is a negative number.

This information is included with the known information. Although the velocity v_{2y} wasn't given in the problem statement, it must—just like for the ball in Example 1.6—be zero at the very top of the trajectory. Last, we have identified y_2 as the desired unknown. This, of course, is not the only unknown in the problem, but it is the one we are specifically asked to find.

ASSESS If you've had a previous physics class, you may be tempted to assign a_{1y} the value -9.8 m/s^2, the free-fall acceleration. However, that would be true only if there is no air resistance on the rocket. We will need to consider the *forces* acting on the rocket during the second half of its motion before we can determine a value for a_{1y}. For now, all that we can safely conclude is that a_{1y} is negative.

FIGURE 1.23 Pictorial representation for the rocket.



Our task in this section is not to *solve* problems—all that in due time—but to focus on what is happening in a problem. In other words, to make the translation from words to symbols in preparation for subsequent mathematical analysis. Modeling and the pictorial representation will be our most important tools.

1.8 Units and Significant Figures

TABLE 1.2 The basic SI units

Quantity	Unit	Abbreviation
time	second	s
length	meter	m
mass	kilogram	kg

Science is based upon experimental measurements, and measurements require *units*. The system of units used in science is called *le Système Internationale d'Unités*. These are commonly referred to as **SI units.** In casual speaking we often refer to *metric units*.

All of the quantities needed to understand motion can be expressed in terms of the three basic SI units shown in TABLE 1.2. Other quantities can be expressed as a combination of these basic units. Velocity, expressed in meters per second or m/s, is a ratio of the length unit to the time unit.

An atomic clock at the National Institute of Standards and Technology is the primary standard of time.

Time

The standard of time prior to 1960 was based on the *mean solar day*. As time-keeping accuracy and astronomical observations improved, it became apparent that the earth's rotation is not perfectly steady. Meanwhile, physicists had been developing a device called an *atomic clock*. This instrument is able to measure, with incredibly high precision, the frequency of radio waves absorbed by atoms as they move between two closely spaced energy levels. This frequency can be reproduced with great accuracy at many laboratories around the world. Consequently, the SI unit of time—the second—was redefined in 1967 as follows:

> One *second* is the time required for 9,192,631,770 oscillations of the radio wave absorbed by the cesium-133 atom. The abbreviation for second is the letter s.

Several radio stations around the world broadcast a signal whose frequency is linked directly to the atomic clocks. This signal is the time standard, and any time-measuring equipment you use was calibrated from this time standard.

Length

The SI unit of length—the meter—was originally defined as one ten-millionth of the distance from the north pole to the equator along a line passing through Paris. There are obvious practical difficulties with implementing this definition, and it was later abandoned in favor of the distance between two scratches on a platinum-iridium bar stored in a special vault in Paris. The present definition, agreed to in 1983, is as follows:

> One *meter* is the distance traveled by light in vacuum during 1/299,792,458 of a second. The abbreviation for meter is the letter m.

This is equivalent to defining the speed of light to be exactly 299,792,458 m/s. Laser technology is used in various national laboratories to implement this definition and to calibrate secondary standards that are easier to use. These standards ultimately make their way to your ruler or to a meter stick. It is worth keeping in mind that any measuring device you use is only as accurate as the care with which it was calibrated.

By international agreement, this metal cylinder, stored in Paris, is the definition of the kilogram.

Mass

The original unit of mass, the gram, was defined as the mass of 1 cubic centimeter of water. That is why you know the density of water as 1 g/cm^3. This definition proved to be impractical when scientists needed to make very accurate measurements. The SI unit of mass—the kilogram—was redefined in 1889 as:

> One *kilogram* is the mass of the international standard kilogram, a polished platinum-iridium cylinder stored in Paris. The abbreviation for kilogram is kg.

The kilogram is the only SI unit still defined by a manufactured object. Despite the prefix *kilo,* it is the kilogram, not the gram, that is the SI unit.

Using Prefixes

We will have many occasions to use lengths, times, and masses that are either much less or much greater than the standards of 1 meter, 1 second, and 1 kilogram. We will do so by using *prefixes* to denote various powers of 10. **TABLE 1.3** lists the common prefixes that will be used frequently throughout this book. Memorize it! Few things in science are learned by rote memory, but this list is one of them. A more extensive list of prefixes is shown inside the front cover of the book.

Although prefixes make it easier to talk about quantities, the SI units are meters, seconds, and kilograms. Quantities given with prefixed units must be converted to SI units before any calculations are done. Unit conversions are best done at the very beginning of a problem, as part of the pictorial representation.

TABLE 1.3 Common prefixes

Prefix	Power of 10	Abbreviation
giga-	10^9	G
mega-	10^6	M
kilo-	10^3	k
centi-	10^{-2}	c
milli-	10^{-3}	m
micro-	10^{-6}	μ
nano-	10^{-9}	n

Unit Conversions

Although SI units are our standard, we cannot entirely forget that the United States still uses English units. Thus it remains important to be able to convert back and forth between SI units and English units. **TABLE 1.4** shows several frequently used conversions, and these are worth memorizing if you do not already know them. While the English system was originally based on the length of the king's foot, it is interesting to note that today the conversion 1 in = 2.54 cm is the *definition* of the inch. In other words, the English system for lengths is now based on the meter!

TABLE 1.4 Useful unit conversions

1 in = 2.54 cm
1 mi = 1.609 km
1 mph = 0.447 m/s
1 m = 39.37 in
1 km = 0.621 mi
1 m/s = 2.24 mph

There are various techniques for doing unit conversions. One effective method is to write the conversion factor as a ratio equal to one. For example, using information in Tables 1.3 and 1.4, we have

$$\frac{10^{-6}\text{ m}}{1\ \mu\text{m}} = 1 \quad \text{and} \quad \frac{2.54\text{ cm}}{1\text{ in}} = 1$$

Because multiplying any expression by 1 does not change its value, these ratios are easily used for conversions. To convert 3.5 μm to meters we compute

$$3.5\ \cancel{\mu\text{m}} \times \frac{10^{-6}\text{ m}}{1\ \cancel{\mu\text{m}}} = 3.5 \times 10^{-6}\text{ m}$$

Similarly, the conversion of 2 feet to meters is

$$2.00\ \cancel{\text{ft}} \times \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} \times \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}} \times \frac{10^{-2}\text{ m}}{1\ \cancel{\text{cm}}} = 0.610\text{ m}$$

Notice how units in the numerator and in the denominator cancel until only the desired units remain at the end. You can continue this process of multiplying by 1 as many times as necessary to complete all the conversions.

Assessment

As we get further into problem solving, you will need to decide whether or not the answer to a problem "makes sense." To determine this, at least until you have more experience with SI units, you may need to convert from SI units back to the English units in which you think. But this conversion does not need to be very accurate. For example, if you are working a problem about automobile speeds and reach an answer of 35 m/s, all you really want to know is whether or not this is a realistic speed for a car. That requires a "quick and dirty" conversion, not a conversion of great accuracy.

TABLE 1.5 Approximate conversion factors. Use these for assessment, not in problem solving.

1 cm	$\approx \frac{1}{2}$ in
10 cm	$\approx$ 4 in
1 m	$\approx$ 1 yard
1 m	$\approx$ 3 feet
1 km	$\approx$ 0.6 mile
1 m/s	$\approx$ 2 mph

TABLE 1.5 shows several approximate conversion factors that can be used to assess the answer to a problem. Using 1 m/s $\approx$ 2 mph, you find that 35 m/s is roughly 70 mph, a reasonable speed for a car. But an answer of 350 m/s, which you might get after making a calculation error, would be an unreasonable 700 mph. Practice with these will allow you to develop intuition for metric units.

NOTE These approximate conversion factors are accurate to only one significant figure. This is sufficient to assess the answer to a problem, but do *not* use the conversion factors from Table 1.5 for converting English units to SI units at the start of a problem. Use Table 1.4.

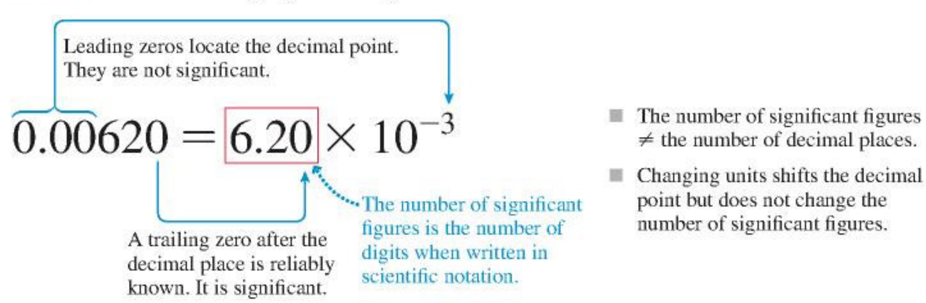
Significant Figures

It is necessary to say a few words about a perennial source of difficulty: significant figures. Mathematics is a subject where numbers and relationships can be as precise as desired, but physics deals with a real world of ambiguity. It is important in science and engineering to state clearly what you know about a situation—no less and, especially, no more. Numbers provide one way to specify your knowledge.

If you report that a length has a value of 6.2 m, the implication is that the actual value falls between 6.15 m and 6.25 m and thus rounds to 6.2 m. If that is the case, then reporting a value of simply 6 m is saying less than you know; you are withholding information. On the other hand, to report the number as 6.213 m is wrong. Any person reviewing your work—perhaps a client who hired you—would interpret the number 6.213 m as meaning that the actual length falls between 6.2125 m and 6.2135 m, thus rounding to 6.213 m. In this case, you are claiming to have knowledge and information that you do not really possess.

The way to state your knowledge precisely is through the proper use of **significant figures.** You can think of a significant figure as being a digit that is reliably known. A number such as 6.2 m has *two* significant figures because the next decimal place—the one-hundredths—is not reliably known. As **FIGURE 1.24** shows, the best way to determine how many significant figures a number has is to write it in scientific notation.

FIGURE 1.24 Determining significant figures.



What about numbers like 320 m and 20 kg? Whole numbers with trailing zeros are ambiguous unless written in scientific notation. Even so, writing 2.0×10^1 kg is tedious, and few practicing scientists or engineers would do so. In this textbook, we'll adopt the rule that *whole numbers always have at least two significant figures,* even if one of those is a trailing zero. By this rule, 320 m, 20 kg, and 8000 s each have two significant figures, but 8050 s would have three.

Calculations with numbers follow the "weakest link" rule. The saying, which you probably know, is that "a chain is only as strong as its weakest link." If nine out of ten links in a chain can support a 1000 pound weight, that strength is meaningless if the tenth link can support only 200 pounds. Nine out of the ten numbers used in a calculation might be known with a precision of 0.01%; but if the tenth number is poorly known, with a precision of only 10%, then the result of the calculation cannot possibly be more precise than 10%.

TACTICS BOX 1.6

Using significant figures

1. When multiplying or dividing several numbers, or taking roots, the number of significant figures in the answer should match the number of significant figures of the *least* precisely known number used in the calculation.
2. When adding or subtracting several numbers, the number of decimal places in the answer should match the *smallest* number of decimal places of any number used in the calculation.
3. Exact numbers are perfectly known and do not affect the number of significant figures an answer should have. Examples of exact numbers are the 2 and the π in the formula $C = 2\pi r$ for the circumference of a circle.
4. It is acceptable to keep one or two extra digits during intermediate steps of a calculation, to minimize rounding error, as long as the final answer is reported with the proper number of significant figures.
5. Examples and problems in this textbook will normally provide data to either two or three significant figures, as is appropriate to the situation. **The appropriate number of significant figures for the answer is determined by the data provided.**

Exercises 38–39

NOTE Be careful! Many calculators have a default setting that shows two decimal places, such as 5.23. This is dangerous. If you need to calculate 5.23/58.5, your calculator will show 0.09 and it is all too easy to write that down as an answer. By doing so, you have reduced a calculation of two numbers having three significant figures to an answer with only one significant figure. The proper result of this division is 0.0894 or 8.94×10^{-2}. You will avoid this error if you keep your calculator set to display numbers in *scientific notation* with two decimal places.

EXAMPLE 1.10 Using significant figures

An object consists of two pieces. The mass of one piece has been measured to be 6.47 kg. The volume of the second piece, which is made of aluminum, has been measured to be $4.44 \times 10^{-4}\ \text{m}^3$. A handbook lists the density of aluminum as $2.7 \times 10^3\ \text{kg/m}^3$. What is the total mass of the object?

SOLVE First, calculate the mass of the second piece:

$$\begin{aligned} m &= (4.44 \times 10^{-4}\ \text{m}^3)(2.7 \times 10^3\ \text{kg/m}^3) \\ &= 1.199\ \text{kg} = 1.2\ \text{kg} \end{aligned}$$

The number of significant figures of a product must match that of the *least* precisely known number, which is the two-significant-figure density of aluminum. Now add the two masses:

$$\begin{array}{r} 6.47\ \text{kg} \\ +\ 1.2\ \ \text{kg} \\ \hline 7.7\ \ \text{kg} \end{array}$$

The sum is 7.67 kg, but the hundredths place is not reliable because the second mass has no reliable information about this digit. Thus we must round to the one decimal place of the 1.2 kg. The best we can say, with reliability, is that the total mass is 7.7 kg.

Proper use of significant figures is part of the "culture" of science and engineering. We will frequently emphasize these "cultural issues" because you must learn to speak the same language as the natives if you wish to communicate effectively. Most students know the rules of significant figures, having learned them in high school, but many fail to apply them. It is important to understand the reasons for significant figures and to get in the habit of using them properly.

TABLE 1.6 Some approximate lengths

	Length (m)
Altitude of jet planes	10,000
Distance across campus	1000
Length of a football field	100
Length of a classroom	10
Length of your arm	1
Width of a textbook	0.1
Length of a fingernail	0.01

TABLE 1.7 Some approximate masses

	Mass (kg)
Small car	1000
Large human	100
Medium-size dog	10
Science textbook	1
Apple	0.1
Pencil	0.01
Raisin	0.001

Orders of Magnitude and Estimating

Precise calculations are appropriate when we have precise data, but there are many times when a very rough estimate is sufficient. Suppose you see a rock fall off a cliff and would like to know how fast it was going when it hit the ground. By doing a mental comparison with the speeds of familiar objects, such as cars and bicycles, you might judge that the rock was traveling at "about" 20 mph.

This is a one-significant-figure estimate. With some luck, you can distinguish 20 mph from either 10 mph or 30 mph, but you certainly cannot distinguish 20 mph from 21 mph. A one-significant-figure estimate or calculation, such as this, is called an **order-of-magnitude estimate.** An order-of-magnitude estimate is indicated by the symbol ~, which indicates even less precision than the "approximately equal" symbol ≈. You would say that the speed of the rock is $v \sim 20$ mph.

A useful skill is to make reliable estimates on the basis of known information, simple reasoning, and common sense. This is a skill that is acquired by practice. Many chapters in this book will have homework problems that ask you to make order-of-magnitude estimates. The following example is a typical estimation problem. TABLES 1.6 and 1.7 have information that will be useful for doing estimates.

EXAMPLE 1.11 Estimating a sprinter's speed

Estimate the speed with which an Olympic sprinter crosses the finish line of the 100 m dash.

SOLVE We do need one piece of information, but it is a widely known piece of sports trivia. That is, world-class sprinters run the 100 m dash in about 10 s. Their *average* speed is $v_{avg} \approx (100\text{ m})/(10\text{ s}) \approx 10$ m/s. But that's only average. They go slower than average at the beginning, and they cross the finish line at a speed faster than average. How much faster? Twice as fast, 20 m/s, would be ≈40 mph. Sprinters don't seem like they're running as fast as a 40 mph car, so this probably is too fast. Let's *estimate* that their final speed is 50% faster than the average. Thus they cross the finish line at $v \sim 15$ m/s.

STOP TO THINK 1.5 Rank in order, from the most to the least, the number of significant figures in the following numbers. For example, if b has more than c, c has the same number as a, and a has more than d, you could give your answer as b > c = a > d.

a. 82 b. 0.0052 c. 0.430 d. 4.321×10^{-10}

SUMMARY

The goal of Chapter 1 has been to learn the fundamental concepts of motion.

GENERAL STRATEGY

Problem Solving

MODEL Make simplifying assumptions.

VISUALIZE Use:

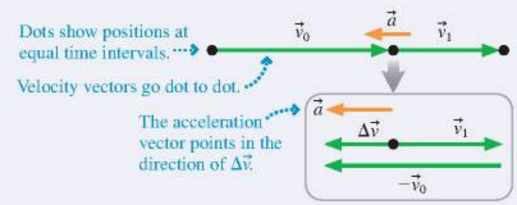
- **Pictorial representation**
- **Graphical representation**

SOLVE Use a **mathematical representation** to find numerical answers.

ASSESS Does the answer have the proper units and correct significant figures? Does it make sense?

Motion Diagrams

- Help visualize motion.
- Provide a tool for finding acceleration vectors.



► These are the *average* velocity and acceleration vectors.

IMPORTANT CONCEPTS

The **particle model** represents a moving object as if all its mass were concentrated at a single point.

Position locates an object with respect to a chosen coordinate system. Change in position is called **displacement.**

Velocity is the rate of change of the position vector $\vec{r}$.

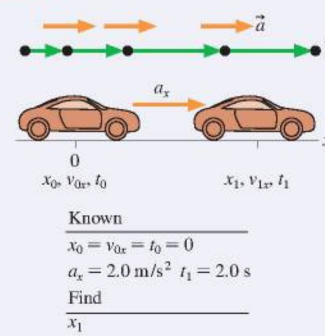
Acceleration is the rate of change of the velocity vector $\vec{v}$.

An object has an acceleration if it

- Changes speed and/or
- Changes direction.

Pictorial Representation

1. Draw a motion diagram.
2. Establish coordinates.
3. Sketch the situation.
4. Define symbols.
5. List knowns.
6. Identify desired unknown.



APPLICATIONS

For **motion along a line:**

- Speeding up: $\vec{v}$ and $\vec{a}$ point in the same direction, v_x and a_x have the same sign.
- Slowing down: $\vec{v}$ and $\vec{a}$ point in opposite directions, v_x and a_x have opposite signs.
- Constant speed: $\vec{a} = \vec{0}$, $a_x = 0$.

Acceleration a_x is positive if $\vec{a}$ points right, negative if $\vec{a}$ points left. The sign of a_x does *not* imply speeding up or slowing down.

Significant figures are reliably known digits. The number of significant figures for:

- **Multiplication, division, powers** is set by the value with the fewest significant figures.
- **Addition, subtraction** is set by the value with the smallest number of decimal places.

The appropriate number of significant figures in a calculation is determined by the data provided.

TERMS AND NOTATION

motion
translational motion
trajectory
motion diagram
model
particle
particle model
position vector, $\vec{r}$
scalar
vector
displacement, $\Delta\vec{r}$
zero vector, $\vec{0}$
time interval, Δt
average speed
average velocity, $\vec{v}$
average acceleration, $\vec{a}$
position-versus-time graph
pictorial representation
representation of knowledge
SI units
significant figures
order-of-magnitude estimate

CONCEPTUAL QUESTIONS

1. How many significant figures does each of the following numbers have?
   a. 0.73 b. 7.30 c. 73 d. 0.073
2. How many significant figures does each of the following numbers have?
   a. 290 b. 2.90×10^4 c. 0.0029 d. 2.90
3. Is the particle in FIGURE Q1.3 speeding up? Slowing down? Or can you tell? Explain.

FIGURE Q1.3

4. Does the object represented in FIGURE Q1.4 have a positive or negative value of a_x? Explain.
5. Does the object represented in FIGURE Q1.5 have a positive or negative value of a_y? Explain.

FIGURE Q1.4

FIGURE Q1.5

6. Determine the signs (positive, negative, or zero) of the position, velocity, and acceleration for the particle in FIGURE Q1.6.

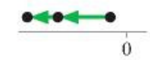


FIGURE Q1.6

7. Determine the signs (positive, negative, or zero) of the position, velocity, and acceleration for the particle in FIGURE Q1.7.

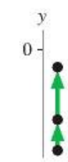


FIGURE Q1.7

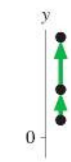


FIGURE Q1.8

8. Determine the signs (positive, negative, or zero) of the position, velocity, and acceleration for the particle in FIGURE Q1.8.

EXERCISES AND PROBLEMS

Exercises

Section 1.1 Motion Diagrams

1. | A car skids to a halt to avoid hitting an object in the road. Draw a basic motion diagram, using the images from the video, from the time the skid begins until the car is stopped.
2. | A rocket is launched straight up. Draw a basic motion diagram, using the images from the video, from the moment of liftoff until the rocket is at an altitude of 500 m.
3. | You are watching a jet ski race. A racer speeds up from rest to 70 mph in just a few seconds, then continues at a constant speed. Draw a basic motion diagram of the jet ski, using images from the video, from 10 s before reaching top speed until 10 s afterward.

Section 1.2 Models and Modeling

4. | a. Write a paragraph describing the particle model. What is it, and why is it important?
   b. Give two examples of situations, different from those described in the text, for which the particle model is appropriate.
   c. Give an example of a situation, different from those described in the text, for which it would be inappropriate.

Section 1.3 Position, Time, and Displacement

Section 1.4 Velocity

5. | You drop a soccer ball from your third-story balcony. Use the particle model to draw a motion diagram showing the ball's position and average velocity vectors from the time you release the ball until the instant it touches the ground.
6. | A baseball player starts running to the left to catch the ball as soon as the hit is made. Use the particle model to draw a motion diagram showing the position and average velocity vectors of the player during the first few seconds of the run.
7. | A softball player slides into second base. Use the particle model to draw a motion diagram showing his position and his average velocity vectors from the time he begins to slide until he reaches the base.

Section 1.5 Linear Acceleration

8. | a. FIGURE EX1.8 shows the first three points of a motion diagram. Is the object's average speed between points 1 and 2 greater than, less than, or equal to its average speed between points 0 and 1? Explain how you can tell.
   b. Use Tactics Box 1.3 to find the average acceleration vector at point 1. Draw the completed motion diagram, showing the velocity vectors and acceleration vector.

FIGURE EX1.8

FIGURE EX1.9

9. | FIGURE EX1.9 shows five points of a motion diagram. Use Tactics Box 1.3 to find the average acceleration vectors at points 1, 2, and 3. Draw the completed motion diagram showing velocity vectors and acceleration vectors.

10. ‖ **FIGURE EX1.10** shows two dots of a motion diagram and vector $\vec{v}_1$. Copy this figure, then add dot 3 and the next velocity vector $\vec{v}_2$ if the acceleration vector $\vec{a}$ at dot 2 (a) points up and (b) points down.

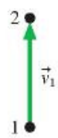


FIGURE EX1.10

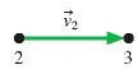


FIGURE EX1.11

11. ‖ **FIGURE EX1.11** shows two dots of a motion diagram and vector $\vec{v}_2$. Copy this figure, then add dot 4 and the next velocity vector $\vec{v}_3$ if the acceleration vector $\vec{a}$ at dot 3 (a) points right and (b) points left.
12. | A speed skater accelerates from rest and then keeps skating at a constant speed. Draw a complete motion diagram of the skater.
13. | A car travels to the left at a steady speed for a few seconds, then brakes for a stop sign. Draw a complete motion diagram of the car.
14. | A goose flies toward a pond. It lands on the water and slides for some distance before it comes to a stop. Draw the motion diagram of the goose, starting shortly before it hits the water and assuming the motion is entirely horizontal.
15. | You use a long rubber band to launch a paper wad straight up. Draw a complete motion diagram of the paper wad from the moment you release the stretched rubber band until the paper wad reaches its highest point.
16. | A roof tile falls straight down from a two-story building. It lands in a swimming pool and settles gently to the bottom. Draw a complete motion diagram of the tile.
17. | Your roommate drops a tennis ball from a third-story balcony. It hits the sidewalk and bounces as high as the second story. Draw a complete motion diagram of the tennis ball from the time it is released until it reaches the maximum height on its bounce. Be sure to determine and show the acceleration at the lowest point.

Section 1.6 Motion in One Dimension

18. ‖ **FIGURE EX1.18** shows the motion diagram of a drag racer. The camera took one frame every 2 s.

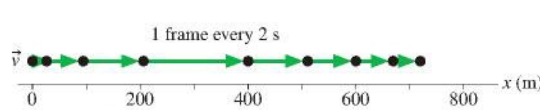


FIGURE EX1.18

a. Measure the x-value of the racer at each dot. List your data in a table similar to Table 1.1, showing each position and the time at which it occurred.

b. Make a position-versus-time graph for the drag racer. Because you have data only at certain instants, your graph should consist of dots that are not connected together.

19. | Write a short description of the motion of a real object for which **FIGURE EX1.19** would be a realistic position-versus-time graph.

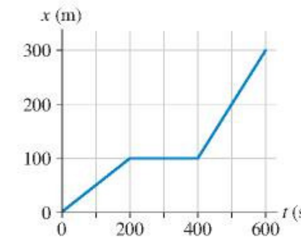


FIGURE EX1.19

20. | Write a short description of the motion of a real object for which **FIGURE EX1.20** would be a realistic position-versus-time graph.

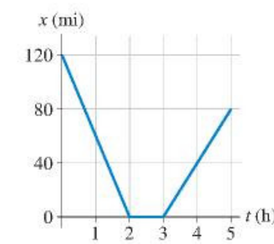


FIGURE EX1.20

Section 1.7 Solving Problems in Physics

21. ‖ Draw a pictorial representation for the following problem. Do *not* solve the problem. The light turns green, and a bicyclist starts forward with an acceleration of 1.5 m/s^2. How far must she travel to reach a speed of 7.5 m/s?
22. ‖ Draw a pictorial representation for the following problem. Do *not* solve the problem. What acceleration does a rocket need to reach a speed of 200 m/s at a height of 1.0 km?

Section 1.8 Units and Significant Figures

23. | How many significant figures are there in the following values?
    a. 0.05×10^{-4} b. 0.00340
    c. 7.2×10^4 d. 103.00
24. ‖ Convert the following to SI units:
    a. 8.0 in b. 66 ft/s
    c. 60 mph d. 14 in^2
25. | Convert the following to SI units:
    a. 75 in b. 3.45×10^6 yr
    c. 62 ft/day d. 2.2×10^4 mi^2
26. ‖ Using the approximate conversion factors in Table 1.5, convert the following SI units to English units *without* using your calculator.
    a. 30 cm b. 25 m/s
    c. 5 km d. 0.5 cm
27. | Using the approximate conversion factors in Table 1.5, convert the following to SI units *without* using your calculator.
    a. 20 ft b. 60 mi
    c. 60 mph d. 8 in

28. | Compute the following numbers, applying the significant figure rules adopted in this textbook.
  a. 33.3×25.4 b. $33.3 - 25.4$
  c. $\sqrt{33.3}$ d. $333.3 \div 25.4$

29. | Perform the following calculations with the correct number of significant figures.
  a. 159.31×204.6 b. $5.1125 + 0.67 + 3.2$
  c. $7.662 - 7.425$ d. $16.5/3.45$

30. | Estimate (don't measure!) the length of a typical car. Give your answer in both feet and meters. Briefly describe how you arrived at this estimate.

31. | Estimate the height of a telephone pole. Give your answer in both feet and meters. Briefly describe how you arrived at this estimate.

32. | BIO Estimate the average speed with which the hair on your head grows. Give your answer in both m/s and μm/hour. Briefly describe how you arrived at this estimate.

33. | BIO Motor neurons in mammals transmit signals from the brain to skeletal muscles at approximately 25 m/s. Estimate how long in ms it takes a signal to get from your brain to your hand.

Problems

For Problems 34 through 43, draw a complete pictorial representation. **Do *not* solve these problems or do any mathematics.**

34. | A Porsche accelerates from a stoplight at $5.0\ \text{m/s}^2$ for five seconds, then coasts for three more seconds. How far has it traveled?

35. | A jet plane is cruising at 300 m/s when suddenly the pilot turns the engines up to full throttle. After traveling 4.0 km, the jet is moving with a speed of 400 m/s. What is the jet's acceleration as it speeds up?

36. | Sam is recklessly driving 60 mph in a 30 mph speed zone when he suddenly sees the police. He steps on the brakes and slows to 30 mph in three seconds, looking nonchalant as he passes the officer. How far does he travel while braking?

37. | You would like to stick a wet spit wad on the ceiling, so you toss it straight up with a speed of 10 m/s. How long does it take to reach the ceiling, 3.0 m above?

38. | A speed skater moving across frictionless ice at 8.0 m/s hits a 5.0-m-wide patch of rough ice. She slows steadily, then continues on at 6.0 m/s. What is her acceleration on the rough ice?

39. | Santa loses his footing and slides down a frictionless, snowy roof that is tilted at an angle of 30°. If Santa slides 10 m before reaching the edge, what is his speed as he leaves the roof?

40. | A motorist is traveling at 20 m/s. He is 60 m from a stoplight when he sees it turn yellow. His reaction time, before stepping on the brake, is 0.50 s. What steady deceleration while braking will bring him to a stop right at the light?

41. | A car traveling at 30 m/s runs out of gas while traveling up a 10° slope. How far up the hill will the car coast before starting to roll back down?

42. || Ice hockey star Bruce Blades is 5.0 m from the blue line and gliding toward it at a speed of 4.0 m/s. You are 20 m from the blue line, directly behind Bruce. You want to pass the puck to Bruce. With what speed should you shoot the puck down the ice so that it reaches Bruce exactly as he crosses the blue line?

43. || David is driving a steady 30 m/s when he passes Tina, who is sitting in her car at rest. Tina begins to accelerate at a steady $2.0\ \text{m/s}^2$ at the instant when David passes. How far does Tina drive before passing David?

Problems 44 through 48 show a motion diagram. For each of these problems, write a one or two sentence "story" about a *real object* that has this motion diagram. Your stories should talk about people or objects by name and say what they are doing. Problems 34 through 43 are examples of motion short stories.

44. |

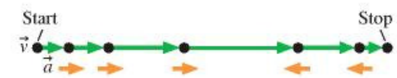


FIGURE P1.44

45. |

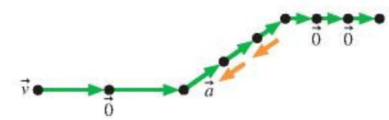


FIGURE P1.45

46. |

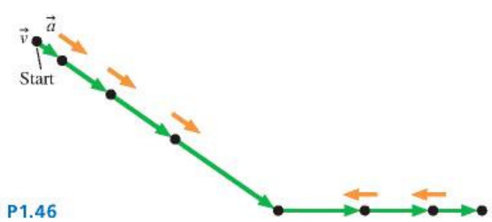


FIGURE P1.46

47. |

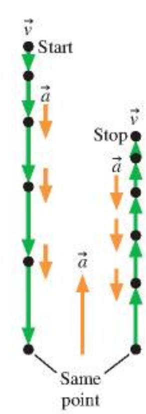


FIGURE P1.47

48. |

FIGURE P1.48

Problems 49 through 52 show a partial motion diagram. For each:

a. Complete the motion diagram by adding acceleration vectors.
b. Write a physics *problem* for which this is the correct motion diagram. Be imaginative! Don't forget to include enough information to make the problem complete and to state clearly what is to be found.
c. Draw a pictorial representation for your problem.

49.

FIGURE P1.49

50.

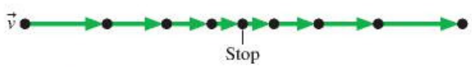


FIGURE P1.50

51.

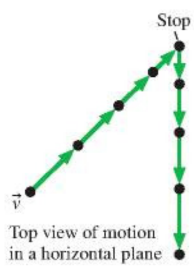


FIGURE P1.51

52.

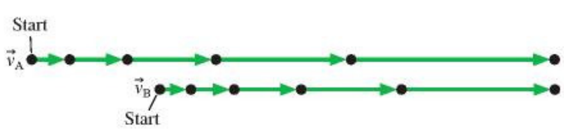


FIGURE P1.52

53. | A regulation soccer field for international play is a rectangle with a length between 100 m and 110 m and a width between 64 m and 75 m. What are the smallest and largest areas that the field could be?

54. | As an architect, you are designing a new house. A window has a height between 140 cm and 150 cm and a width between 74 cm and 70 cm. What are the smallest and largest areas that the window could be?

55. || A 5.4-cm-diameter cylinder has a length of 12.5 cm. What is the cylinder's volume in SI units?

56. | BIO An intravenous saline drip has 9.0 g of sodium chloride per liter of water. By definition, $1\ \text{mL} = 1\ \text{cm}^3$. Express the salt concentration in kg/m^3.

57. || The quantity called *mass density* is the mass per unit volume of a substance. What are the mass densities in SI units of the following objects?

a. A 215 cm^3 solid with a mass of 0.0179 kg.
b. 95 cm^3 of a liquid with a mass of 77 g.

58. | FIGURE P1.58 shows a motion diagram of a car traveling down a street. The camera took one frame every 10 s. A distance scale is provided.

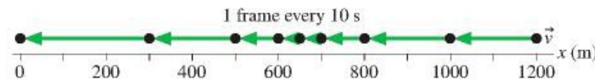


FIGURE P1.58

a. Measure the x-value of the car at each dot. Place your data in a table, similar to Table 1.1, showing each position and the instant of time at which it occurred.
b. Make a position-versus-time graph for the car. Because you have data only at certain instants of time, your graph should consist of dots that are not connected together.

59. | Write a short description of a real object for which FIGURE P1.59 would be a realistic position-versus-time graph.

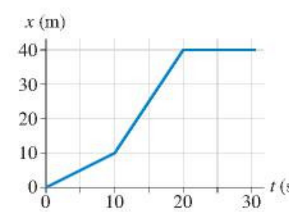


FIGURE P1.59

60. | Write a short description of a real object for which FIGURE P1.60 would be a realistic position-versus-time graph.

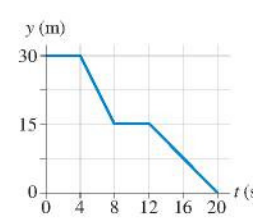


FIGURE P1.60